“Numbers”

A Math and Thinking Challenge

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Published by eiPie Innovations

http://eipie.com

Printed in US by Instantpublisher.com

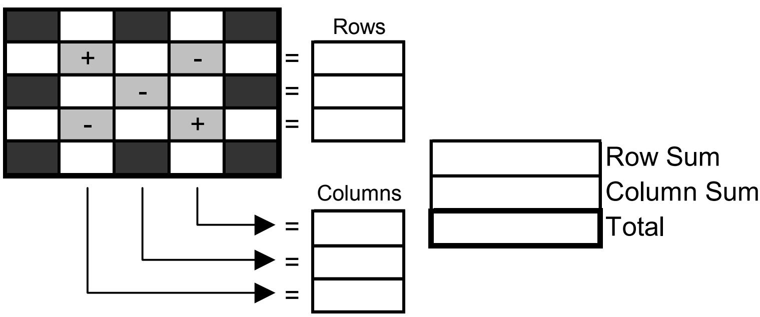
ISBN: 978-0-578-00945-2

Cover image of Earth is courtesy of the Image Science & Analysis Laboratory, NASA Johnson Space Center. (http://eol.jsc.nasa.gov, AS17-148-22727, 12/7/1972)

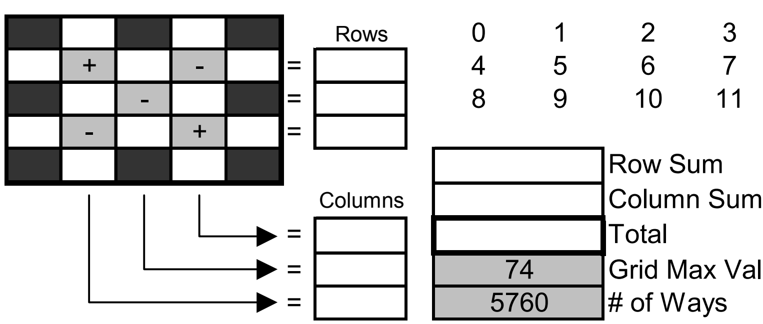
Contents

Introduction

The game of "Numbers"[1] was developed to provide an entertaining way to practice basic mathematics. To jump right in with a simple example, consider the figure below. If the grid's empty cells are filled with ones, each equation evaluated (rows, left-to-right; columns, top-to-bottom), and the results summed, the grid has a value of four.



While there are several ways "Numbers" can be implemented, it is presented in this book in the challenge format. The objective is to take each of the provided numbers, 0 through 11 in the problem below, and insert them into the twelve blank locations with the goal of maximizing the grid value. The basic rules of math apply. Specifically, division by zero is not allowed and



[1] Patent pending

proper order-of-operations must be followed (evaluate any multiplications or divisions in an equation, working left-to-right or top-to-bottom as appropriate; then, in the same manner, evaluate any additions or subtractions to determine the equation's value). The grids may seem a little intimidating at first, but go ahead, pencil in the numbers, evaluate the rows and columns to determine the grid total, and then go back and make adjustments aimed at increasing that total.

After solving any of the five-by-five grids, a little side challenge is to show why there is the specified number of ways to obtain the maximum value. There are 5760 ways in the previous example, but with a few grids there are almost a million different correct number arrangements. There is, however, no need to be concerned that so many right answers indicate a lack of challenge. With nearly 480 million possible arrangements of the twelve numbers, all the grids require some mental effort!

"Numbers" is entertaining because it involves thinking that goes beyond the basics needed to do a series of math problems. The thinking needed is the same type of thinking that provides a delightful challenge to just about any kid with a little bit of money in a candy store. There is pleasure in getting more for our money rather than less, but as we get beyond the childhood years, the tendency is to become a little lazy and let the force of habit take precedence over explicit rational thinking. Simple situations are not an issue – we will buy item X at store Z for 25% off, rather than pay full price at store Y – but if a situation involves some complexity, we do not always counter that complexity with thought. How many vehicle users ever explicitly consider whether is it worth driving an extra 2 miles to save 5 cents per gallon to fill their tank?

Just as a kid in a candy store has limits, everyone has limits on their resources of time, talent, and money. We also know that given two people with different resource levels, the one with more will not necessarily be more effective in life than the one with less. Often, effective use is of greater importance than available quantity (one kid exits the candy store with a nice bag of candy, another comes out with a fancy box that turns out to not have much inside). Between personal resource limits that constrain what we can do, and a world where many critical resources are in limited supply, it is clear that encouraging explicit rational thinking relative to the use of those resources can be nothing but beneficial.

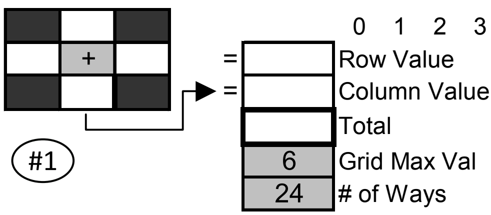
How is it that "Numbers" can help develop this desirable type of thinking? The key is that it drives the participants to evaluate quantitatively the plusses and minuses associated with the various options, and figure out how to get the greatest overall benefit. For example, in the simple grid shown previously, there are cells that get:

1) summed twice (thus providing good locations to place large numbers)
2) summed only once
3) subtracted once
4) subtracted twice (good locations to place small numbers)

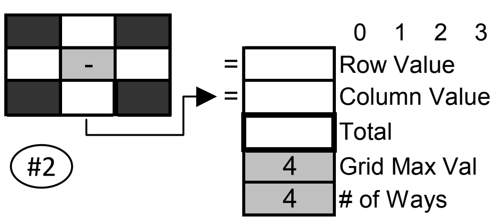
With an understanding of how much impact a number in a given position will have on the grid value, it is possible to proceed with confidence in utilizing the numbers. With simple grids, as in the above example, one can quickly find an optimum number arrangement. As functions other than addition and subtraction are incorporated, and/or the grid gets larger, the difficulty level can increase substantially. As a result, the same basic format, with some variation in focus, can be used to provide challenges for everyone from grade school to graduate school.

Group 1: Three-by-Three Grids

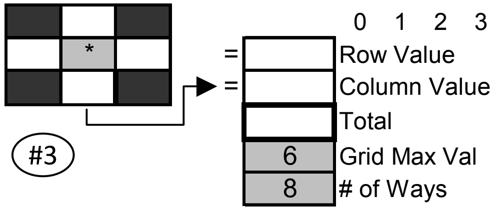
The simplest three-by-three grid is shown below. The single math function is addition, and it does not make any difference where the four integers (0, 1, 2, 3) are placed. Each value gets summed once, and only once, and the only possible grid value is six.

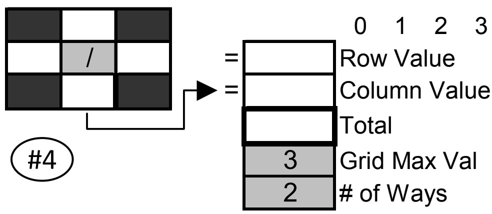


The next simplest grid of this size is obtained by replacing the plus sign by a minus sign. With that change, it is readily apparent that getting the maximum possible grid value means placing the two larger values at the top and left where they are implicitly added, and the two smaller values at the bottom and right where they are subtracted.



By switching the math function first to multiplication and then division, one obtains the next two three-by-three grids.





These four three-by-three grids do not present much of a challenge, but they do serve to convey the concept of arranging numbers so as to obtain the greatest possible grid value. The next grids, each with five rows and five columns, will increase the level of mental exercise.

Group 2: Five-by-Five Grid – Addition

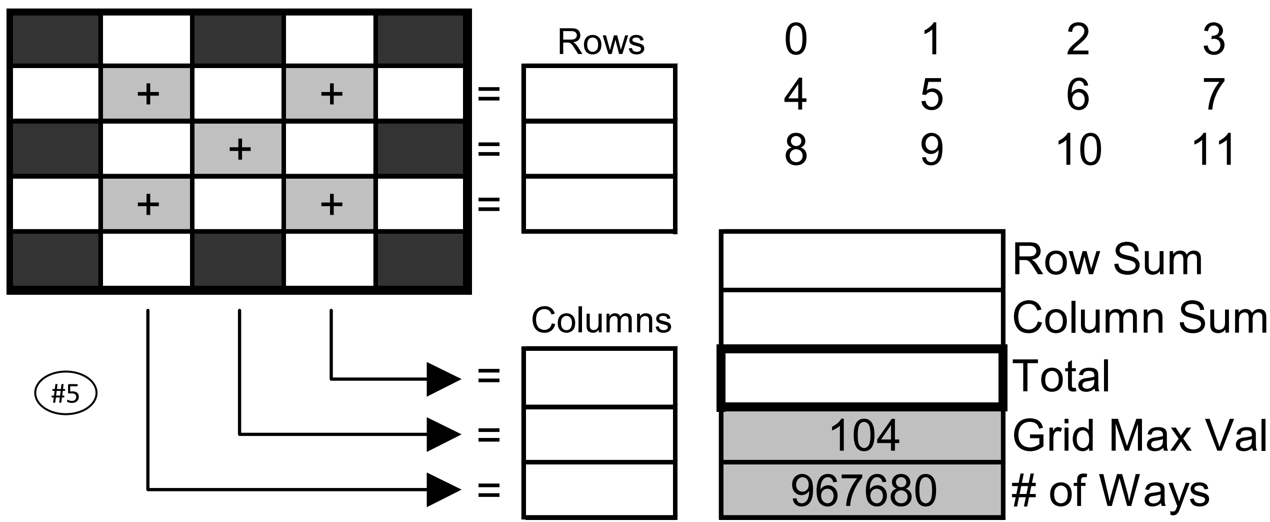
With the larger grid size, but using only addition, the solution is just a little more involved than was the case with the smaller grid. There are now twelve empty cells and, naturally, twelve integers (0 - 11) to be appropriately placed in those cells. Unlike the three-by-three addition grid, however, the arrangement of numbers does matter in the pure addition five-by-five grid.

While some may allow that a mental challenge can be entertaining, most probably wonder why, other than for a school assignment, anyone would be motivated to evaluate the equations in a five-by-five grid. Even with schoolwork, many don’t bother and many others require a parental push. Comments such as, “I don’t know why I have to take this class. I’m never going to use this stuff,” are commonplace. Such students should not be so sure. Whether working at a job, doing things at home, coaching an athletic team, or participating in countless other activities, opportunities exist to make better use of available resources.

Consider the situation of a small manufacturing business. The employees have diverse skill levels and the machines on which they work have different reliability levels. To maximize output, how should the employees be assigned to the machines? The fastest workers *always* seem to get a lot done, so should they be put on the machines that tend to breakdown the most?

The thinking involved in making such worker assignments is the same type of thinking involved in solving a “Numbers” challenge. The decision maker will not always explicitly do the calculations, but the concepts behind a rational decision do entail math from arithmetic up through function maximization. Fundamentally, “Numbers” is an example of the application of math in the quantitative evaluation of a situation, and the making of decisions aimed at optimal use of resources.

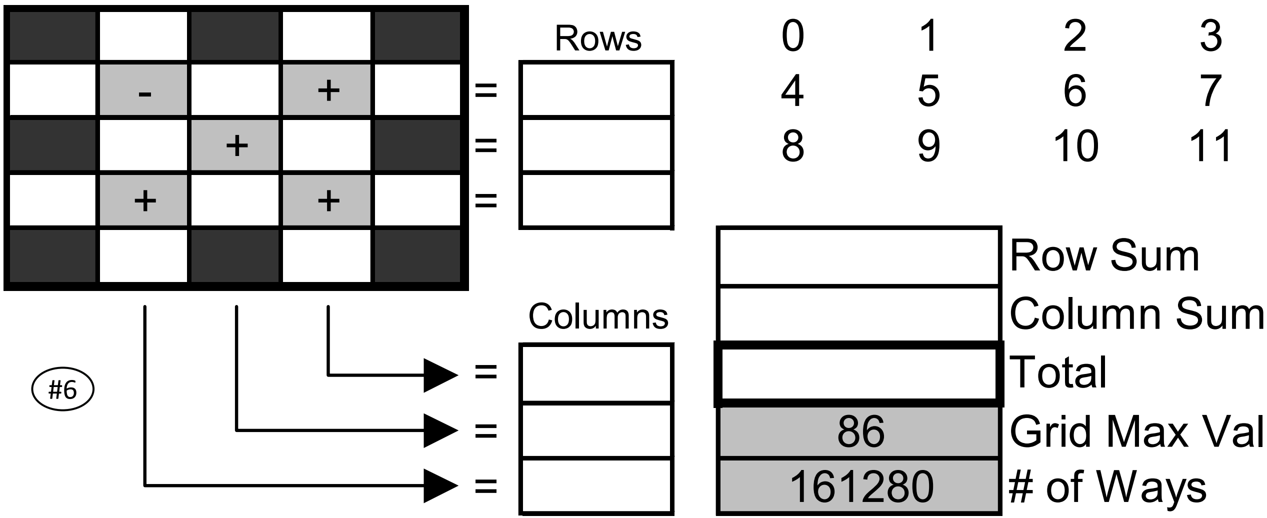
#5



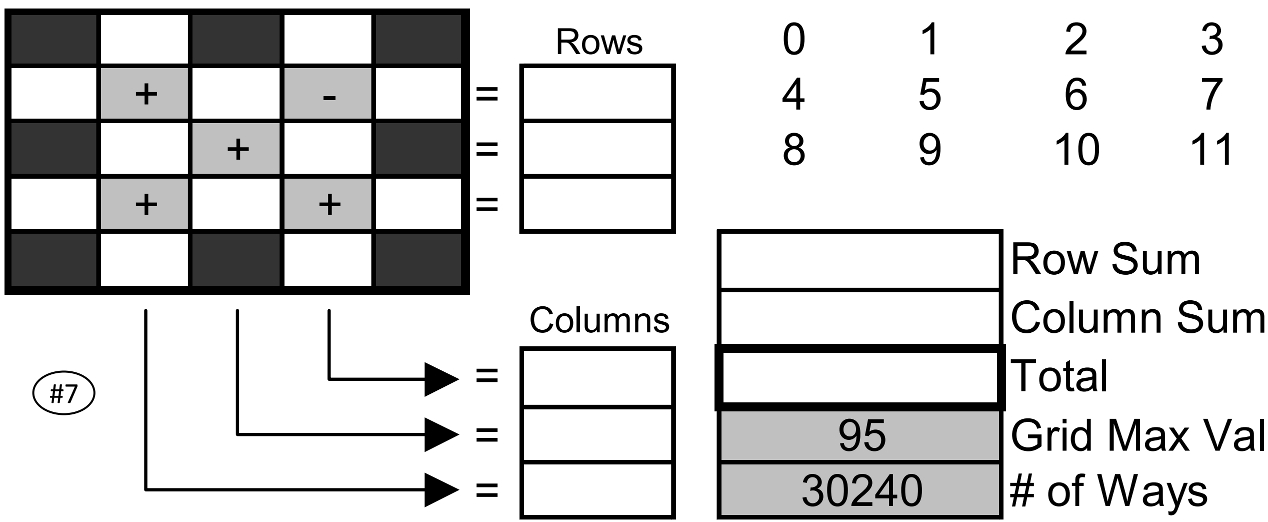
Rows
Columns
+
=
0 1 2 3
4 5 6 7
8 9 10 11
Row Sum
Column Sum
Total
104
Grid Max Val
967680
# of Ways

Group 3: Five-by-Five Grids – Addition & Subtraction

Including subtraction enables a number of different five-by-five grids and begins to generate some more interesting challenges (grids 6-24). The presence of subtraction is analogous to including costs in an evaluation process. Whether in a real world situation, or solving a “Numbers” challenge, it is hard to argue with the strategy of minimizing costs (negative terms) and maximizing benefits (positive terms).

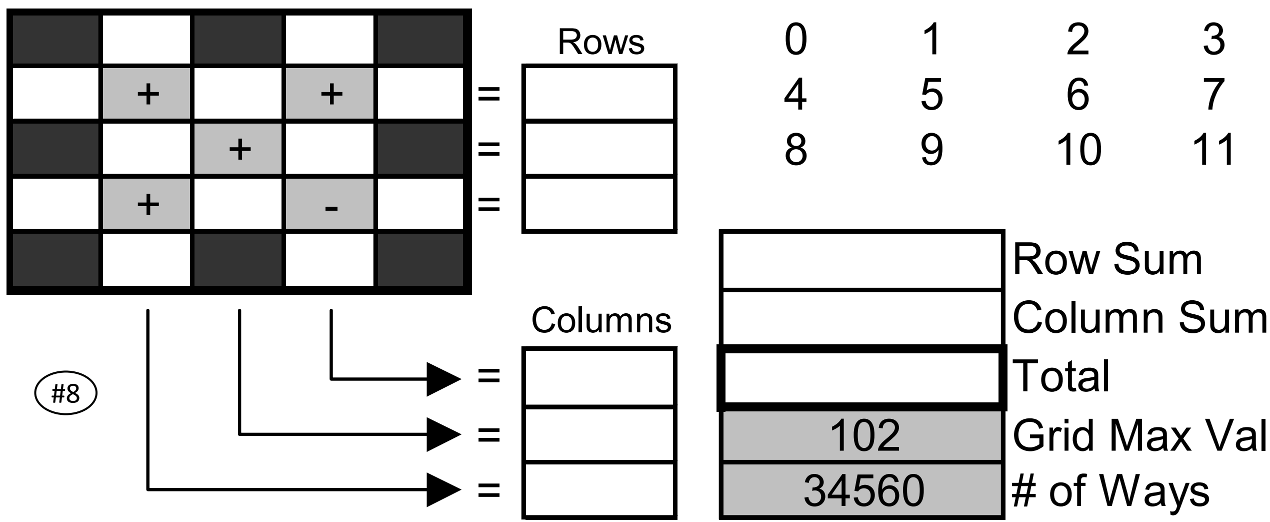

-
+
+
+
+
=
=
=
Rows
Columns
=
=
=
0 1 2 3
4 5 6 7
8 9 10 11
Row Sum
Column Sum
Total
86
Grid Max Val
161280
# of Ways

#6



#7
Rows
Columns
0 1 2 3
4 5 6 7
8 9 10 11
Row Sum
Column Sum
Total
95
Grid Max Val
30240
# of Ways

#8


+ + + + -
=
=
=
Rows
Columns
=
=
=
0 1 2 3
4 5 6 7
8 9 10 11
Row Sum
Column Sum
Total
102 Grid Max Val
34560 # of Ways